Modern Type Theories and Linguistic Semantics

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This talk – two parts

I. Modern Type Theories: brief introduction

- Historical development, basics, meta-theory, …
- Applications (formalisation, verification and semantics)

II. MTTs applied to linguistic semantics

- Several issues with examples
- Rich typing, propositions-as-types, signatures and proofs

Studying type theory and MTT-semantics, I've collaborated with many, only mentioning a few (not all!):

- Adams, Callaghan, Goguen, Pollack (type theory & proof assistants)
- Soloviev, Xue and Y. Luo (coercive subtyping)
- Chatzikyriakidis (MTT-semantics), Asher (linguistic coercions), Lungu (signatures), Maclean (subtype univ) and Shi (adjectives in Chinese)

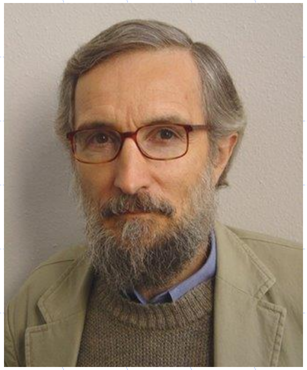
Part I. Modern Type Theories

Historical development of type theory

- Russell's ramified type theory (1925)
  - Paradoxes in naïve set theory
  - Zermelo: axiomatic set theory
  - Russell: ramified type theory ("axiom of reducibility")
- Ramsey (1926)
  - Logical v.s. semantic paradoxes
  - Impredicativity is circular, but not vicious.
- Church's simple type theory (1940)
  - Formal system based on λ-calculus
  - Higher-order logic with simple types (e, t, $e \rightarrow t$, …)

Modern Type Theories

- Martin-Löf has introduced/employed
  - Dependent/inductive types, type universes
  - Judgements with contexts, definitional equality
  - Curry-Howard principle of propositions-as-types
- Dependent types: "types segmented by indexes"
  - List ➔ Vect(n) with n:Nat (lists of length n)
- Examples of MTTs:
  - Predicative TTs:
    - Martin-Löf's intensional type theory MLTT [1973, …]
      (non-standard FOL – strong sum Σ as existential quantifier; Agda)
  - Impredicative TTs (cf, Christian's talk on F, "smallest" impr type sys):
    - CC [Coquand & Huet 1988] and CIC_p (HOL; Coq/Lean)
    - UTT [Luo 1990, 1994] (HOL; Lego/Plastic)



UTT = MLTT + CC

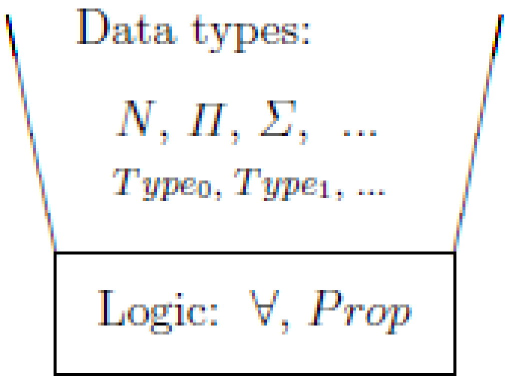
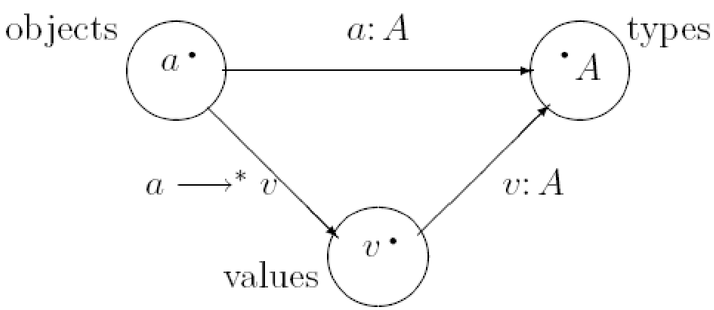


Fig. 1. The type structure in UTT.



Example: A = Nat, a = 3+4, v = 7.
(c.f., verificationistic meaning theory)

- UTT [Luo 90,94] has nice meta-theoretic properties
  - Goguen's PhD thesis on "Typed Operational Semantics" (1994)
  - Strong normalisation, which implies, e.g., logical consistency.

Π-types and $\forall$-props: examples of dependent types

- $\Pi x{:}A.B(x)$ is the collection of functions "from A to B" such that …:

  $\{ f \in A \rightarrow \cup_{a\in A} B(a) \mid \forall a{\in}A.\ f(a){\in}B(a) \}$

- Similarly, universal quantification:

$$\frac{\Gamma \vdash A\ type \quad \Gamma,\ x{:}A \vdash P : Prop}{\Gamma \vdash \forall x{:}A.B : Prop}$$

$$\frac{\Gamma \vdash A\ type \quad \Gamma,\ x{:}A \vdash B\ type}{\Gamma \vdash \Pi x{:}A.B\ type}$$

$$\frac{\Gamma,\ x{:}A \vdash b : B}{\Gamma \vdash \lambda x{:}A.b : \Pi x{:}A.B}$$

$$\frac{\Gamma \vdash f : \Pi x{:}A.B \quad \Gamma \vdash a : A}{\Gamma \vdash f(a) : [a/x]B}$$

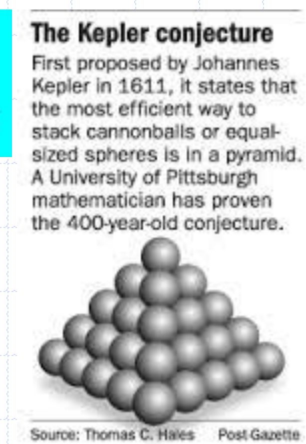
$$\frac{\Gamma,\ x{:}A \vdash b : B \quad \Gamma \vdash a : A}{\Gamma \vdash (\lambda x{:}A.b)(a) = [a/x]b : [a/x]B}$$

  Note: Prop is a type, an "impredicative universe" – formation of propositions is "circular" (e.g., $\forall X{:}Prop.X : Prop$)

- Π-polymorphism (example of uses):
  - small : $\Pi A{:}CN.\ (A \rightarrow Prop)$
  - small(Elephant) : Elephant→Prop
  - small(Mouse) : Mouse→Prop

Type theory based proof technology

- Proof assistants based on type theories
  - MTT-based: ALF/Agda, Coq, Lean, Lego, NuPRL, Plastic, …
  - HOL-based: Isabelle, HOL, …
- Applications of proof assistants
  - Math: formalisation of mathematics
    - 4-colour theorem (Coq), Kepler conjecture (Isabelle)
    - Homotopy type theory [HoTT 2013] (Coq/Agda)
  - Computer Science:
    - program verification and advanced programming
    - Coq applied to verifications [Pierce et al. 2018]
  - Computational Linguistics
    - NL reasoning based on MTT-sem (Coq) [Chatzikyriakidis-Luo 2016]



Part II. MTTs in Linguistic Semantics

Type-Theoretical Semantics

- Montague Semantics (Montague 1930–1971)
  - Dominating in linguistic semantics since 1970s
  - Set-theoretic, using simple type theory as intermediate
- Research on rich typing in NL semantics
  - Ranta (MLTT), Bekki (subsystem of MLTT), Retoré (system F), …
  - Rich typing (type dependency etc.): Asher, Cooper, Grudzińska, …
- MTT-semantics: formal semantics in modern type theories
  - Ranta (1994): formal semantics in Martin-Löf's type theory
  - Luo (2009). Type-Theoretical Semantics with Coercive Subtyping. SALT20.
  - Chatzikyriakidis and Luo. Formal Semantics in Modern Type Theories. Wiley/ISTE, 2020.
  - Luo. Modern Type Theories: Their Development and Applications. Tsinghua University Press. (In Chinese; to appear)

Some features/work in MTT-semantics

- Copredication
  - Example: The lunch was delicious but took forever.
  - Linguistic phenomenon studied by many (Pustejovsky, Asher, Cooper, Retoré, …)
  - Dot-types in MTTs [Luo 2009, Xue & Luo 2012, Chatzikyriakidis & Luo 2018]
  - C.f. talk by Wang later.
- Linguistic coercions via coercive subtyping [Asher & Luo (S&B12)]
- Dependent event types [Luo & Soloviev (WoLLIC17)]
- Propositional forms of judgements [Xue et al (NLCS18)]
- MTT-sem in $MLTT_h$ (MLTT+HoTT's logic) [Luo (LACompLing18)]
- Subtype universes [Maclean & Luo 2021]

Today, we shall consider several (other) issues in MTT-semantics.

Rich typing (1): adjectival modification

- CNs as types [Mönnich 1985, Sundholme 1986, Ranta 1994]
- Adjectival modification
  - [Chatzikyriakidis & Luo 2013,17,20; Luo, Shi & Xue 2022]

Classical classification	**Example**	**Characterisation**	**MTT-semantics**
intersective	black cat	Adj(N) ➔ N & Adj	$\Sigma x{:}Cat.black(x)$
subsective	small elephant	Adj(N) ➔ N	$small : \Pi A{:}CN.\ A \rightarrow Prop$
privative	fake gun	Adj(N) ➔ $\neg$N	$\Sigma x{:}G.fake(G,x)$ with $G=G_R+G_F$
non-committal	alleged criminal	Adj(N) ➔ nothing	$H_{h,Adj} : Prop \rightarrow Prop$

Rich typing (2): subtyping

- Simple example for subtyping

  A human talks. Paul is a handsome man. Does Paul talk?

  Yes, because paul : $\Sigma(Man,handsome) \leq Man \leq Human$.

  ➔ Subtyping is crucial for MTT-semantics.

- Coercive subtyping
  - Developed for general applications of MTTs (proof dev etc.)
    - [Luo 1996, Luo, Soloviev & Xue 2012, Xue 2013, Lungu & Luo 2018]
    - Note: Traditional subtyping is inadequate for MTTs (eg, canonicity fails)
    - c.f., Tao's talk in the first session
  - Useful mechanism for basic/advanced modelling in MTT-sem
- Subtype universes [Maclean & Luo 2021]
  - Bounded quantification: $\Pi X \leq A.B(X)$ or $\forall X \leq A.P(X)$

Propositions as types

- Principle of propositions as types (PaT)
  - P true ←→ p : P for some p
  - Also called "Curry-Howard correspondence":
    - Curry & Feys (1958) for propositional logic
    - Howard (1969) for first-order logic
- Decidability – necessary for PaT logic
  - "P true" v.s. "p : P": the latter has p (proof candidate).
    - "P true" is undecidable. (Intuitively, infinitely many proof candidates.)
    - "p : P" should be decidable. (Our systems are finitely-presented.)
  - Type checking in MTTs is decidable.
    - Eg, UTT is decidable [Goguen 1994]: strong normalisation→decidability
    - Counter-example: Martin-Löf's <u>extensional</u> TT [ML84] is undecidable.

Signatures: mechanism to assume constants

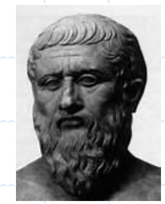
- Signatures in type theory
  - Edinburgh Logical Framework [Harper, Honsell & Plotkin 1993]
  - Adding signatures with membership entries:

$$\Gamma \vdash a : A \ \rightarrow\ \Gamma \vdash_\Delta a : A$$

  where $\Delta = c_1 : A_1, \ldots, c_n : A_n$ (c_i being <u>constants</u>, not variables).
- Signatures in MTT-semantics [Luo 2014]
  - In semantics, (partial) "possible worlds" can be adequately represented as signatures (not contexts in type theory).
  - Subtype entries ($A \leq_c B$) and manifest/"definitional" entries ($c \sim a : A$) for semantic modelling.
  - Preservation of nice properties [Lungu & Luo 2018]

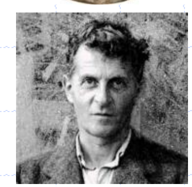
Meaning theories and NL reasoning

- Theories of meaning
  - Meaning is reference ("referential theory")
    - Word meanings are things (abstract/concrete) in the world.
    - c.f., Plato, …



  - Meaning is concept ("internalist theory")
    - Word meanings are ideas in the mind.
    - c.f., Aristotle, …, Chomsky.

  - Meaning is use ("use theory")
    - Word meanings are understood by their uses.
    - c.f., Wittgenstein, …, Dummett, Brandom.



- MTT-semantics is proof-theoretic as well as "model-theoretic"
  - MTTs are defined by rules and have use theory of meaning [Martin-Löf 84]
  - MTT-semantics implemented in existing proof assistants for NL reasoning. (E.g., application of Coq [Chatzikyriakidis & Luo 2016, 2020])

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